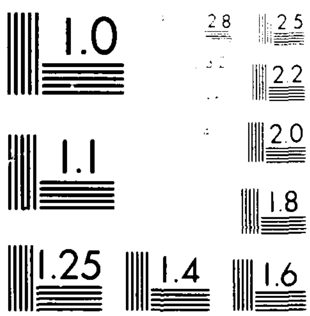
AD-A113 353 OHIO STATE UNIV COLUMBUS DEPT OF GEODETIC SCIENCE A--ETC F/G 8/5
STUDY OF GEODETIC AND GEOPHYSICAL PARAMETERS.(U)
DEC 81 U A UOTILA F19628-79-C-0075
UNCLASSIFIED 324 AFGL-TR-82-0018 NL

1 OF 1
AD A
113353

END
DATE
FILMED
05-82
DTIC


1.0
2.8
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6

(12)

AFGL-TR-82-0018

AD A113 353

STUDY OF GEODETIC AND GEOPHYSICAL PARAMETERS

URHO A. UOTILA

DEPARTMENT OF GEODETIC SCIENCE
AND SURVEYING
THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION

DECEMBER 1981

FINAL REPORT FOR PERIOD-1 DECEMBER 1978 to 30 SEPTEMBER 1981

AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AFG, MASSACHUSETTS 01731

Qualified requestors may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-82-0018	2. GOVT ACCESSION NO. AD-A113353	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) STUDY OF GEODETIC AND GEOPHYSICAL PARAMETERS		5. TYPE OF REPORT & PERIOD COVERED Final Report-1 December 1978 30 September 1981
		6. PERFORMING ORG. REPORT NUMBER Report No. 324
7. AUTHOR(s) URHO A. UOTILA		8. CONTRACT OR GRANT NUMBER(s) F19628-79-C-0075
9. PERFORMING ORGANIZATION NAME AND ADDRESS Dept. of Geodetic Science and Surveying The Ohio State University - 158 Neil Avenue Columbus, Ohio 43210		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62101F 760003AL
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory Hanscom AFB, Massachusetts 01731 Contract Monitor: George Hadgigeorge/LW		12. REPORT DATE December 1981
		13. NUMBER OF PAGES 33
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE Unclassified

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION ST... ENT (of ... abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY ... TES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

reference frames
nutation
computational methods in geodesy
polar motion
surface representation
gravity reference network
inertial navigation
mass point modeling

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report summarizes the research work carried out from December 1, 1978 through September 30, 1981 on an AFGL project directed towards the study of geodetic and geophysical parameters. The final report summarizes the work distributed earlier in twelve scientific and two internal reports. The main areas of research were: reference frames, nutation and polar motion; prediction, estimation and other computational methods in geodesy; general surface representation module designed for geodesy; U.S. gravity reference network; positional errors in inertial navigation and mass point modeling.

DD FORM 1 JAN 73 1473

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

i/ii

FOREWORD

This report was prepared by Urho A. Uotila, Professor, Department of Geodetic Science and Surveying at The Ohio State University, under Air Force Contract No. F19628-79-C-0075, OSURF Project No. 210-711715. This is a final report of the contract covering the time period 1 December 1978 to 30 September 1981. The contract has been administered by the Air Force Geophysics Laboratory, Air Force Systems Command, Hanscom AFB, Massachusetts with Mr. Bela Szabo, Contract Monitor.

The author thanks all of those who have participated in the research under the contract. Special mentioning should be made about the excellent contributions of Drs. Meissl, Moritz, Schwarz, and Sünkel and Mr. Krieg. The author acknowledges the cooperation and support given and expresses his appreciation to the Contract Monitor, Mr. Bela Szabo, for the stimulating technical and scientific discussions.

TABLE OF CONTENTS

TABLES

FIGURES

Study of Geodetic and Geophysical Parameters

1. Introduction

This is a final report on research done under the contract mentioned in the foreword. The research has previously reported in twelve scientific reports and two internal reports. In the following a summary of the scientific work, done under the contract, will be summarized and some work, which has not been presented as a scientific report, will be discussed. The scientific reports produced under the contract will be freely quoted without quotation marks. Subject matter will be discussed in this report rather than the statement of work defined under various line items in the contract.

2. Reference Frames, Nutation and Polar Motion

2.1 Geodetic Reference Frames

Conceptional problems are of importance for a definition of precise reference systems to be used for very accurate geodetic purposes, down to the centimeter level (part 1 in 10^8 or better). In 1974 the IAU Colloquium No. 26 was held on Reference Coordinate Systems for Earth Dynamics in Torun, Poland and it was devoted entirely to this topic. Since that there has been several articles and reports on the subject matter which is receiving increasing attention by theorists and those working with data. Geodesists and geophysicists are beginning to explore, debate and adopt reference systems, therefore it is particularly appropriate to have a general review of geodetic reference frames at this time. It has been done by Moritz in his report entitled: "Concepts in Geodetic Reference Frames" [Moritz, 1979].

For geodetic, geodynamics and astronomic purposes, two basic coordinate systems are needed: an inertial system and an earth-fixed system. First Moritz discusses basic physical, conventional and average systems. There he deals with inertial systems from three points of view: 1) classical theory; 2) relativistics aspects; and 3) practical realization. After detailed discussions of these subjects, the concept of celestial

pole, which plays a fundamental role in a precise definition of precession, nutation and polar motion, is reviewed in detail.

Next Moritz devotes a section of the report to tidal effects, which should be removed from the coordinates. In the final section of the report he is comparing various definitions of terrestrial reference systems. He concludes, for future requirements, that for a more precise definition, at the centimeter level, satellite laser and interferometer methods determining Cartesian coordinates x, y and z seem to be better suited than astronomical coordinates, Φ Λ, whose accuracy can hardly be essentially improved beyond the present level. He also states that the best approach seems to be that systematic effects such as tides and plate motions, should be modeled as far as possible and that residual motions and similar effects should be taken into account by averaging over a certain number of given stations. In his report Moritz touches most of the issues in establishing a terrestrial reference frame, but does not advocate any particular solutions. Importance of this report might be reflected by an unusual large number of requests received for a copy of this report.

2.2 Theories of Nutation and Polar Motions

As mentioned above two basic coordinate systems, inertial and earth-fixed, are needed. Those two systems are related to each other through procession, nutation, and polar motion.

In order to relate to these two systems in a precise fashion, highly sophisticated observational methods such as doppler, lunar laser, and VLBI observations must be available. They are not the only ones needed, but also very accurate theories, which take into account the elasticity of the earth's mantel, as well as effects due to liquid core are essential. A clear understanding of these theories is a prerequisite also for practical work in this field.

Mortiz makes a systematic review of the theories in his report [Moritz, 1980]. He presents the basic principles in a rather detailed manner, and states that the report is also suitable as an introductory text even for scientists with little or no previous knowledge of the field.

The report [Moritz, 1980] is restricted to those theories which regard the earth as a rigid body, such as Kinoshita's recent theory, or as a purely elastic solid, such as McClure's work, or as a body consisting of a rigid mantle and a liquid core, the so-called Poincaré model.

Emphasis in the report is on the treatment of an elastic earth on the basis of Liouville's equation, leading to a systematic theory of polar motion, precession, and nutation of various axes (rotating axis, angular momentum axis, figure axis, and the so-called celestial reference pole), and on the eigenvalue problem for rotation, leading to a similar theory for a rigid earth and for the Poincaré model.

None of the models discussed in the report is fully realistic, but each contains important features which form a basis indispensable for understanding, and even for practially and numerically treating, a more realistic model consisting of an elastic mantle and a liquid core. The considerations of models of the latter kind, which is a complicated and difficult subject, is treated by Moritz in his other report [Moritz, 1981].

The first theoretical treatment of an earth model with an elastic mantle and a liquid core has been given already by Poincaré [1910], but only to the extent to show that a nearly diurnal free wobble exists. His treatment is sketchy and incomplete; his approach has not been followed by later authors. No quantitative results are given.

Only in 1949, Harold Jeffreys took up the problem again and used a variational method, although a somewhat simplified one, to get preliminary numerical values. A perfected version of the variational approach of Jeffreys and Vicente in 1957 gave the first numerical results that are so detailed and accurate as to come close to modern standards, although the underlying earth models were rather simplified.

The variational approach of Jeffreys and Vicente has not been pursued since. In 1961 Molodensky writes: "Jeffreys and Vicente applied a variational method which made it difficult to have a clear notion about the degree of approximation to the exact solution of the problem. I attempted to reproduce this theory and to make it more exact, but owing to numerous obscurities I had to give up this method."

Thus, Molodensky developed a method of his own, based on a spherical approximation of the equations of elasticity for the mantle (and, possibly, for a solid inner core) and on a hydrodynamical theory for the liquid core. He got numerical results which are excellent even by present standards; furthermore, this approach--generalized, modified, simplified--has been the starting point of practically all subsequent work.

Unfortunately, the classical papers by Jeffreys-Vicente and Molodensky are very difficult to read, because of "certain expository difficulties in both". On the other hand, according to Moritz, they are important enough to deserve careful study.

Moritz gives in his report [Moritz, 1981] an introduction to the work of Jeffreys, Molodensky and subsequent developments. He interrelates various approaches using a basic set of parameters (essentially the work of Molodensky) and relates it to other variables such as employed by Jeffreys. The various coordinate systems used by different authors is reduced by Moritz to two basic systems, the nutation frame (Jeffreys, Smith, Wahr) and the body frame (Molodensky, Shen-Mansinha), and he gives the relations between the systems.

Moritz states that the most difficult part of Molodensky's treatment is the hydrodynamic theory of the core. In order to clarify the essential features and to supply motivations which are missing in Molodensky's presentation, Moritz presents a "pedestrian's" version using a simplified model of a homogeneous core. Moritz states that this model, although inadequate from a practical point of view, is best suited for a first approach; it permits an instructive comparison of the methods of Jeffreys and Molodensky with the classical theory of Poincaré (for a rigid mantle). Moritz also shows the relation of Molodensky's theory to modern treatments of the core in terms of toroidal oscillations (Smith, Shen and Mansinha, Wahr).

It is expected that these two reports [Moritz, 1980 and 1981] will be basic reference sources in the area of nutation and polar motion for many scientists in the future as his report on reference frames [Moritz, 1979] has proven to be in that area.

3. Prediction and Estimation

3.1 Approximation Procedures for Computations of Values of Covariance Functions

Modern technology has created highly sophisticated instruments which enable us to obtain different kinds of geodetic data with increasing accuracy at a speed never known before. Probably the only technique capable of processing all these data simutaneously and in a constant way is the method of least-squares collocation.

The central role in collocation is played by the model covariance function which resembles somehow the main features of the earth's gravitational field. The property of the kernel function forces it to be harmonic outside some internal sphere; consequently, its eigen-functions are the spherical harmonics. Homogeneity and isotropy postulates make it dependent on only two essential variables: the spherical distance between two points and the product of the corresponding geocentric radii of these points.

All geodetic data are functionals of the disturbing potential, therefore all covariances between geodetically relevant quantities can be derived by means of the covariance propagation law. This again, is the reason why closed analytical expressions of the covariance function are an absolute necessity. Tscherning and Rapp have derived closed expressions on the basis of different degree variance models in 1974 and give covariance expressions for geoidal height, gravity anomaly and deflection of the vertical, most commonly used in geodetic applications. Tscherning extended this work and derived covariance expressions for second order derivatives of the anomalous potential in 1976. His very elegant and convenient subroutine COVAX has been widely applied by the geodetic community.

However, the number of problems, for which the application of COVAX becomes expensive as far as computer time is involved, increases steadily. All these expensive problems involve the numerical integration of the covariance function resulting in a large number of covariance computations. Two typical examples: a) the prediction of mean gravity anomalies from different kinds of data, and b) satellite-to-satellite tracking (especially

in the low-low mode). In both cases multifold integrations of the covariance function are involved (in the first case integrations over "rectangular' areas on the sphere, in the second case integrations along the flight paths of two satellites).

For the above reason the possibility of cutting down the computation time of the covariance values by using approximations of the covariance function has been studied by Sünkel under the previous contract. His theoretical investigations indicated that there exists a possibility for this and his preliminary practical calculations encouraged further detailed studies. In his continued research [Sünkel, 1979a], he has found a way, which permits a consistent approximation of all covariance expressions up to and including second-order derivatives of the disturbing potential. Basically, his method consists of choosing a regular rectangular grid with respect to the cosine of the spherical distance, $t_j = \cos\psi_j$, and with respect to the squared ratio, s, between the radius of the Bjerhammar sphere, R_B, and some radius $r > R_B$, $s_i = (R_B / r_i)^2$, and storing all covariances corresponding to these grid points on some file. According to Sünkel, all desired covariance expressions can be derived by a simple differentiation-interpolation procedure. He states that the accuracy of the so obtained covariances can be made arbitrary high by selecting an appropriate small grid spacing.

The procedure presented by Sünkel [1979a] provides the same covariances for a user as conveniently as COVAX, but about 10 times faster. The price to be paid for this is a requirement for mass storage. However, computer development statistics show clearly that the amount of available mass storage increases much more than the speed-up of calculation time. Moreover, since large scale applications of collocation (for which the procedures described in the report is designed) are by their own nature restricted to large computer systems with a large central core capacity available, the need for storage can no longer be considered an essential drawback. It is believed that the presented approximation procedure will be especially valuable for collocation problems which involve the integration of the covariance function. A FORTRAN IV program listing together with sample inputs and outputs are included in the report [Sünkel, 1979a].

3.2 Comparison of Various Kind of Base Formulas Used for Prediction

Essential features of any mathematical method might be studied best if extreme cases were considered. As far as interpolation, filtering, prediction, or even least-squares collocation are concerned, an infinite homogeneous set of regularly distributed data presents itself as an ideal candidate for such kind of studies. Without restriction of generality one can assume a unit distance between neighboring data points, such that the data are located at the places of the cardinal numbers on the real line: this is why Sünkel has used the term "cardinal" in the title of his report [Sünkel, 1981b]. The second term in the title "interpolation", does not require a further interpretation, even though his study goes beyond that.

The goal of Sünkel's study was to investigate the response of the choice of various base (covariance) functions onto the interpolated (predicted) function based on an infinite homogeneous data set. In particular, the family of splines, the Gaussian function, the $\sin\pi x/\pi x$ function, the Hirvonen covariance functions, and the Bjerhammar interpolation functions are considered by Sünkel in his report [Sünkel, 1981b].

In the first chapter Sünkel deals with the whole family of spline base functions, starting with the zero degree spline base function (= step function) and finally arriving at the highest degree spline, whose corresponding sampling function equals the $\sin\pi x/\pi x$ function.

In the second chapter, relations between Gaussian functions of various correlation lengths and the corresponding spline functions of certain degree are established. Deviations of these two functions from each other are estimated.

Structural similarities between spline interpolation and least-squares interpolation are discussed in chapter 3. The finite support of splines and their ability to fit and replace any kind of covariance (= base) function leads Sünkel to the conclusion that splines of high and odd degree may advantageously be used as base functions for the solution of least-squares prediction problems.

In the fourth chapter, Sünkel investigated a virtually completely different interpolation method, which has been advocated by Bjerhammar and others. Sünkel found an interesting and remarkable relation of that interpolation method to spline functions of odd degree.

Next Sünkel investigated the impact of data noise and correlation length onto the sampling functions and the prediction error. He found that there is a very strong relation between the correlation length of the underlying covariance function, the sampling rate (distance between data points), the data noise, and the maximum prediction error. This finding is of importance of all concerned about data collection. Its impact might be considerable on practical gravity field sampling problems.

It is astonishing and remarkable that such strong ties exist between all of the base functions investigated. A beautiful interplay between the physical and the frequency domain is given a dominant role in all investigations peformed by Sünkel in his report [Sünkel, 1981b].

<u>3.3 Prediction of the Gravity Disturbance Vector at High Altitudes</u>

Under certain circumstances, the gravity disturbance vector is wanted at high altitudes. The accuracy of the determination of the vector is depending upon the computational method used, data available, and the accuracy of the data.

In order to get an idea of the obtainable accuracies of predicted vectors as a function of distribution and accuracy of the data on the surface of the earth, Sünkel conducted simulation studies using basic data sets consisting of nine different 5'x5', 15'x15', 1°x1° and 5°x5° mean anomaly sets with two sets of accuracies and reports his results in report: "Feasibility Study for the Prediction of Gravity Disturbance Vector in High Altitudes" [Sünkel, 1981a].

Sünkel uses two methods in his study, namely the least-squares collocation and the integral solution. It was hoped that he could have used the programs developed by himself earlier [Sünkel, 1979b], but he found out that even though a complete and consistent prediction algorithm was available in the above mentioned report, it was necessary for him to design a new and strictly problem oriented procedure. The prediction

algorithm given in the earlier report [Sünkel, 1979b] is designed for a general case of a heterogeneous set of irregularly distributed data, which does not allow to take an advantage of symmetries in the data distribution. Since the data sets to be studied were fairly large, more than 2000 data points per set, Sünkel found out that a straight forward least-squares collocation solution would have required excessive computer time as compared to the effort to develop specially tailored programs to these special cases in hand.

Sünkel first develops an optimal algorithm for the collocation solution of the problem in his report [Sünkel, 1981a]. This algorithm is taking advantage of the regular data distribution and it turns out to be up to 64 times faster than the algorithm developed in his earlier report [Sünkel, 1979b]. Then he develops the formulations and algorithms for integral solution, in which he performs the estimation of the representation error in a frequency domain.

After studying the 18 data sets covering the entire earth, Sünkel concludes that the radial component of the gravity disturbance vector can be estimated with an accuracy of one mgal at the altitude of about 50,000 ft. on the basis of the available data sets. The errors in the data sets, particularly those of 5'x5' mean anomalies has to be reduced by some 60% in order to achieve the same accuracy at 30,000 ft. His studies show once more that the prediction error drops quickly with an increasing altitude as far as radial component is concerned.

The situation is considerably different for the prediction errors of the horizontal components of the gravity disturbance vector. Sünkel states that with the best data set used an accuracy of 2.3 mgal can be achieved at 30,000 ft. altitude. He concludes that in order to obtain one mgal accuracy a considerable better representation of the gravity field, than those used in the simulations, is required. He estimates that in the critical region (up to 30° spherical distance from the computation point) the block sizes need to be reduced by a factor of about 2 and overall data accuracy should be increased by about 30% in order to achieve this goal. He gives the results of his estimate to all 18 data sets at five different altitudes.

Sünkel also compares two methods of estimation of gravity disturbance vector, which he presents in the report, namely the least-squares collocation solution and the integral solution. He concludes, the estimations done with these two methods will differ only little, if the gravity coverage is reasonably good.

3.4 Prediction of Free-air Anomalies on Topographically Rough Areas

One of the many applications of least-squares collocation is the prediction of point and mean gravity anomalies based on point gravity anomalies. It is hardly any problem, as long as the terrain is flat within the area of consideration. Free-air anomalies can be processes directly, no additional data reduction seems to be necessary. In mountainous areas the situation changes dramatically; suddenly data reduction becomes indispensable. As it is well know the free-air anomalies are correlated with the local topography and the terrain correction is playing an important role in computation of mean anomalies in mountainous areas.

Sünkel's study [Sünkel, 1981c] aims at an optimal estimation of point and mean anomalies taking into account the concept of a linear correlation between terrain-corrected free-air anomalies and topographic height. As a first item Sünkel discusses mean free-air anomalies and the effect of topography on them and then goes to discussions of prediction of free-air anomalies by trend removal. He concludes that the least-squares collocation with parameters presents itself as a very attractive and powerful tool for simultanious estimation of regression parameters and point and/or mean anomalies. Next Sunkel presents an explanation for the regional variation of the regression parameters based on a simplified concept of isostatic compensation. He concludes that if the least-squares adjustment concept were used for estimation of Bouguer gradient the selection of an appropriate block size should be done very carefully, but the determination of Bouguer coefficient through the least-squares collocation is insensitive with respect of the choice of the block size. This finding is of great importance for those whose who have to predict anomalies on mountainous area, however there is still the effect of topography, which should not be forgotton.

Next Sünkel reports about his studies on the effect of terrain to the point and mean anomalies. As it has been known for a long time, the computation of the effect of topography is a very laborious task. Sünkel shows that the effect depends linearly on the terrain variance and it is inversely proportional to the correlation length of the terrain covariance function valid for the considered area. The effect depends weakly on the type of the covariance model. Sünkel concludes from his studies [Sünkel, 1981c], that the total variance of the topography depends on the power spectrum; if the high frequency part of the spectrum has much power, a detailed terrain model is required in order to estimate the variance with a sufficient accuracy. He also states that the terrain correlation length plays a fundamental role in estimations of the mean terrain effect; a short correlation length requires a high sampling rate. He recommends further investigations on the resolution of the digital terrain models for computation of the terrain effect on gravity anomalies. Sünkel's investigations reported here are on basic importance for analyses of terrain effect on upward continued anomalies.

4. A General Surface Representation Module Designed for Geodesy

As mentioned earlier in the report, the modern technology has created highly sophisticated instruments which enable us to obtain data with increasing accuracy and at a speed never known before. In order to make data processing faster, computers have been used, which are designed in such a way as to permit solutions to many different kinds of problems. A relatively small number of functions is provided with any computer systems. It is usually up to users to write their own programs for their required purposes, but rigorously tested subroutines are available in program libraries. A set of programs could be combined to a large module, which would be possibly capable of handling a large set of problems. Such kinds of modules could be very large, such as a satellite orbit prediction module. Users of these modules do not necessarily know the total content of them, but a module is supposed to be intelligent by itself. This means that such a module should be able to check the input for consistency, make adjustments if necessary and assign proper default values to unidentified parameters. It is most interesting and often difficult to split up human decision processes into steps or statements and to translate them into a computer language.

The very complexity of a module usually makes it hard for users to understand and therefore many times these modules are called "black boxes". Sünkel presents that kind of "black box" in his report entitled: "A General Surface Representation Module Designed for Geodesy", [Sünkel, 1979b] and follow-ups with two internal reports: "Manual to the General Surface Representation Module Designed for Geodesy (GSPP)" [Sünkel, 1980a] and "Program Listing for the General Surface Representation Module Designed for Geodesy (GSPP)" [Sünkel, 1980b]. These three reports contain a total of four hundred sixty-five pages of explanations of Sünkel's "black box".

The first report [Sünkel, 1979b] contains the theoretical discussions of the formulas and procedures used in the Geodetic Science Plotting Package (GSPP), which is designed to give graphical representation of data. As it is known, a suitable graphical presentation of data is very useful and revealing on many occasions. Sünkel describes the various algorithms of GSPP used to predict, by number of different methods, a bicubic spline surface from irregularly distributed data and to produce profiles, contour maps and 3-D views of a surface and its first or second derivatives. The least-squares collocation method is not forgotten in the presentation. He also has designed the programs to do complete labeling of graphs. The contour maps can be produced in many map projections defined by the user.

The first internal report [Sünkel, 1980a] provides detailed information on all input parameters, messages and output from the GSPP, a brief description of the subprograms and other details. The second internal report [Sünkel, 1980b] includes the listings of the programs. The users should be made aware that these reports were partly made at The Ohio State University and partly at the Technical University, Graz, Austria. The computers available at these locations are not comparable, but there is supposed to be a version for each computer, however as always in a large package of programs there might be some parts of the programs, which require fine tuning for the computer to be used. These programs are available to qualified users through the sponsor of this research project.

5. Computational Methods in Physical Geodesy

There exists a number of methods used in computational physical geodesy and some new ideas, which have not been examined previously. Meissl examines various methods currently used to approach the problem of determination of the earth's figure and potential from the viewpoint of computational efficiency in his report: "The Use of Finite Elements in Physical Geodesy" [Meissl, 1980]. Methods like collocation, surface layer, buried masspoints, Bjerhammar's method, lead to a fully occupied linear system of equations to be solved. Meissl states that the effort to solve such a system is proportional to N^3, where N is the number of equations. Breakdown due to OSU CPU times exceeding 100 hours occurs at about N = 10,000 (this corresponds to a surface layer solution with 2° x 2° blocks near the equator.

After analizing various methods, Meissl concludes that if the pattern of data and weights shows rotational symmetry, great savings in computation time can be obtained by using techniques based on discrete Fourier transform of block circulant matrices. This has been shown by Colombo in 1980. Meissl feels, after examination of all methods, that Colombo's approach is currently the best one if an essentially non-redundant set of surface data is employed. Such a set is for example, given by 1° x 1° block averages of gravity anomalies. Although the quality of such block averages varies greatly between areas, the assumption of equal weights will not cause too much harm to the estimated parameters according to Meissl, because there is no problem of adjusting redundant data. The system must take what it gets and has no choice to balance poor anamalies against better observations. Of course, a drawback of the method is, as stated by Meissl again, that the accuracy estimates obtained from such a procedure are very problematic.

Meissl continues that similar things may be said about GEOFAST developed by TASC. The asymptotic speed is even proportional to N log N. The gain in speed is paid for by restricting applications to data distributed regularly on a line or within a rather small plane rectangle. Some possible trouble spots are indicated by Meissl in chapter 2. One of them is concerned with transporting a covariance from the sphere to the plane. Harmonicity gets lost thereby. Meissl would like to have

some estimate on the proportionality factor in front of the N log N term estimating the CPU time.

The main trust of the report of Meissl is a feasibility study on the finite element method in physical geodesy which is given in chapters 3 to 6 of the document. According to Meissl this method leads to a sparse set of equations whose solution requires an effort proportional to $N^{3/2}$, where N has the same meaning as above. Unfortunately, the constant of proportionality is large. Meissl estimates that the break even point between the surface layer and finite elements in a global solution is around 2° x 2° blocks. Meissl shows that for small blocks finite elements are faster; for larger ones the surface layer is faster. He estimates that the effort for a global solution based on 1° x 1° gravity anomaly data is at 700 OSU CPU hours. A special technique exploiting the remote zone effect could reduce this to about 250 hours (the surface layer method would require 15,000 hours). An effort of 250 OSU CPU hours is considered too large by Meissl. He states that several reruns would be necessary before a satisfactory choice of weights is found. Although there exist computers, as for example the ILLIAC IV, on the CPU time could be cut by a factor of about 64, the problem appears to be, according to Meissl, too large for one individual researcher or a small research group.

Fortunately, the remote zone effect allows to compute local solutions. In the report Meissl gives an estimate for the calculation of a detailed potential in an equatorial strip (6.5 hours) and in a rectangular area of size 32° x 64° (covering e.g. the conterminous US). In the latter case 30' x 30' data were assumed by Meissl. CPU time was estimated by him at 20 OSU hours. Meissl estimates that by a sophisticated use of the remote zone effect this can probably be lowered to 6 hours. This compares favorably with a surface layer solution requiring about 70 hours.

The beauty of the finite element method is according to Meissl that it does not rely on any reqular pattern of observations and weights. (Regularity could be exploited in the same way as with the other methods. The additional saving in CPU time would, however, not be dramatic.) In

areas where the field shows much detail, smaller elements may be chosen. Redundant data, as for example gravity anomalies plus geoid heights, pose no problem. Disadvantages of the method, according to Meissl, are; harmonicity of the calculated field is only approximate and the programming effort for an efficient computer implementation is considerable.

Meissl found out that the finite element method loses much of its efficiency if the data are not local. Local data are composed of measurements taken in a way that one measurement involves only a single point or a small vicinity of a point. A vicinity of a point is considered small if it contains only a small number of the finite elements in its interior. A more precise definition of the locality of a measurement would be that its contribution to the normal equations must not destroy the sparsity pattern resulting from a field representation by means of finite elements. Meissl gives an example that data obtained by integrating over an unknown orbit are not local and neither are misclosures of large inertial navigation loops. Meissl points out, that there are, however, ways to incorporate orbits at higher altitudes in an efficient way.

The finite element method lends itself to Helmert blocking, or its modern variant, nested dissection. Calculations for subregions (nations, continents) could be delegated. Junction equations could be combined at a higher level, very much in the same way as this is done in continental network adjustment.

In chapter 6 of the report a number of test calculations are performed by Meissl. They were carried out with the following goals and results:

1) To see whether the Ritz-, or the Trefftz-, or the old fashioned least-squares principle should be used. Meissl recommends the latter for the specific needs of physical geodesy.

2) To see whether the use of cubic polynomials is sufficient, or whether quintic or even higher degree polynomials are needed. Meissl reports that cubics are sufficient, quintics are already hopeless from the CPU time point of view.

3) To see whether a certain type of element partition making a best possible use of the "attenuation with altitude effect" can be employed. Meissl finds the outcome satisfactory.

4) To find an appropriate representation of the field in the remote outer space of the earth. Meissl finds that specially designed elements of infinite size and appropriately chosen shape functions perform well in this respect.

5) To get an idea how the observational weights should be balanced against the weights applied to the equations enforcing the approximate fulfillment of Laplace's equation. Meissl finds reasonable weights by experiment, but he states that more insight would be desirable.

6) To see whether a combination solution of surface gravity values and satellite derived harmonics is possible. Meissl finds it possible, but he feels that additional tests are necessary to identify procedures preventing a substantial increase in CPU time.

Meissl carried out the experiments in 2 dimensions in order to save CPU time. He wanted to do small scale 3-dimensional calculations, but there was no time yet to perform them.

Meissl's opinion is that the finite element method has a place in physical geodesy. It is very likely that a proposal by Junkins will be accepted, suggesting to use a finite element representation of a completely known potential for the purpose of rapid recalculation in real time application and also otherwise. Meissl feels that finite elements are also useful to parameterise an unknown potential during an estimation procedure. However, the method must be cultivated somewhat more before a large scale effort is attempted. At the end of the research period covered by this report, Meissl began to look into a hybrid method which combines finite elements with a surface layer of multipoles. Some preliminary statements on this envisioned method are given in chapter 7. According to Meissl another feature which makes finite elements attractive is the possibility to attack in a head-on way free boundary value problems of physical geodesy. Meissl gives some ideas how this could be accomplished in chapter 5.

It is believed that the research reported by Meissl (Meissl, 1981] is fundamental in nature as so many of his previous reports done under the research contracts of the same sponsor.

The results of Meissl's research [Meissl, 1980] show that the use of finite elements in physical geodesy looks promising, therefore a research effort on this subject matter should be continued in the future.

6. U.S. Gravity Reference Network

Accurate values of gravity are required for many purposes, for example, for geodetic and geophysical computations and analyses, therefore it has been an interest of scientists around the world for a long time to provide a homogeneous world wide gravity reference system. The first internationally accepted gravity reference system was known as the "Vienna Gravity System", the "Potsdam Gravity System" was adopted in 1909. It was replaced by a new reference system: "The International Gravity Standardization Net 1971 [I.G.S.N.71]", which was adopted by the International Union of Geodesy and Geophysics at the XVth General Assembly in Moscow, August 1971.

The adjustment of IGSN 71 included 10 absolute gravity measurements, about 1200 pendulum measurements and 23700 gravity meter measurements. The least-squares solution provided the gravity values at 1854 stations, scale factors for 96 gravity meters and drift rates for 26 instruments. Twenty-six of these stations are located in Hawaii, 21 in Alaska and 379 in the continental United States. It is very unlikely that any new world wide simultaneous adjustment of a reference system will be done in the future. Base station networks in the future will be local or continental in nature. This is possible and still the gravity values will be in the same uniform reference system provided that absolute gravity measurements and/or IGSN 71 values are used properly in forming the networks and in their adjustment.

Since the adoption of IGSN 71 many new gravity meter measurements and absolute gravity determinations have been made in the continental United States which could be used for a new reference system. It is a great interest to make an adjustment of the new data and compare the

results with the values published in IGSN 71. This is especially important, because there has been some indications that IGSN 71 could have some scale problems. It is also possible that new, more accurate, measurements could produce more accurate gravity values at the reference stations. For example, the accuracy of absolute determination of gravity values has improved. In IGSN 71 there were only 10 absolute measurements from which three were in the continental United States excluding Alaska. There are now 28 new absolute measurements available with two instruments and over 4500 new gravity meter measurements in a new network in the continental United States excluding Alaska [Krieg, 1981].

In order to do an adjustment of data, the input data must be carefully analized for the quality and for removal of blunders. The second step is to establish proper mathematical models to be used in the adjustment.

In order to establish proper mathematical models, the behavior of instruments used should be carefully studied. This is done in the first part of Krieg's research in his report [Krieg, 1981]. In chapter 2 of his report Krieg discusses construction of LaCoste-Romberg G gravity meter, which was most commonly used in the new gravity meter observations. Special attention was paid to factory calibration of instrument and instrumental errors, such as periodic screw errors.

He also discusses observational techniques used in new observations and problems involved in this area. In the following chapter he gives a short presentation of optimal designs of gravity base station networks.

In the establishment of IGSN 71 two distinctly different mathematical models were used. In one of the models, gravity differences were adjusted and in the other one, dial readings converted to milligal equivalent were used as observed quantities. This latter one allowed to have parameters which were higher order correction terms to the calibration of the gravity meters.

One of the major contributions of Krieg's research is the improvement of the mathematical model to be used for gravity meter observations. Gravity meter measurements enter to the adjustment model as dial units and periodic screw error terms are solved for the gravity meters for which sufficient data exist. He also treats corrections to the calibrations of the instrument as long wave periodic terms.

Gravity meter measurements were obtained from the copies of the original field notes and the absolute measurements were entered as reported by the observers. Krieg made a countless number of adjustments of data in connection of data analyses and in determinations of the significance of some parameters in the mathematical model. The analyses made showed that the new relative gravity measurements were almost free of any large blunders [Krieg, 1981]. However there were some problems in the identification of observation sites, occupied by two or more groups. The accuracies of the measurements were between 15 to 30 microgal depending on which particular instrument was used.

Accuracies of absolute measurements are claimed by the observers to be about 10 microgal. New absolute measurements have been made at 14 sites in the United States. Some of them have been occupied by two or more different absolute apparatuses and some of them have been occupied by the same apparatus twice or more times. These repeated observations gave results, which in most cases agreed within the limits of claimed accuracies, but at some sites these differed by as much as 100 microgal. The analyzed measurements were made by two apparatuses, one developed at the Istituto di Metrologia "G. Colonnetti", Torino, Italy and another one developed at the Air Force Geophysics Laboratory by Hammond. The differences in these absolute measurements have caused one of the problems in the adjustments. It might be more appropriate to estimate the accuracy of the absolute apparatuses to be about 20 to 25 microgal rather than 10 microgal. This higher estimate might include some environmental effects, which are very difficult to estimate.

As mentioned before many adjustments of these data have been made by Krieg [1981], but only three adjustments are discussed here. Each of the three adjustments included all accepted relative gravity measurements, but the first one (Adjustment D) included only absolute measurements made by Hammond, the second one (Adjustment E) included only the absolute measurements made with the Italian apparatus and the third one (Adjustment F) included only absolute measurements made with both apparatuses at four common sites, where the results seemed to be in reasonable agreement. The differences between adjusted gravity values from those three adjustments seemed to indicate some systematic differences, which were related

to magnitude of the gravity values [Krieg, 1981]. The gravity values of the first adjustment (D) and those from the third adjustment (F) seemed to agree with each others better than the values of the secomd adjustment (E) with the results of the third adjustment (F). It is difficult to say if this is significant or not because the network might be too flexible and the absolute measurements were not equally distributed along the gravity range nor east-west locations.

A comparison of the three solutions has been made to the values of those IGSN 71 stations, which are common to both the nets. There were 124 common stations in the nets, from which 51 were "main stations" and the rest of them were in a close vicinity of the main stations. Three of 124 were obviously not common stations even though they had the same IGB identification numbers. Root mean square values, mean values and standard deviations of the differences between the IGSN 71 values and the values obtained from Adjustments D, E and F are given in Table 1. In the same table the corresponding information is also given for the differences between the adjustments of D, E and F, but only for the same stations above.

Table 1

Difference	Root mean square microgal	Mean microgal	Standard Dev. microgal
IGSN 71 - D	47.3	-21.4	42.4
IGSN 71 - E	60.7	- 9.4	60.7
IGSN 71 - F	57.1	+33.7	46.2
E - D	35.4	-12.0	33.4
D - F	63.8	+55.6	32.2
E - F	77.5	+43.1	64.7

The IGSN 71 values and the values of the new adjustments seem to be in a relatively good agreement, much better than 100 microgal, which was the accuracy claimed to the IGSN 71 values. These numbers show that there may be a small systematic difference. Any possible trends have to be analysed from graphs, where the differences are plotted as functions of magnitudes of the gravity values and longitudes of the station.

In Figure 1 the differences between IGSN 71 and Adjustment D are plotted as functions of the magnitude of gravity values. There are few stations, where differences are high, for example at IGB 12172 K San Francisco it is -165 μgal, but other San Francisco stations had much smaller differences, i.e., 127172 A +26 μgal and 12172 -14 μgal. There seems to be a problem in local ties in the IGSN 71 net. There is a similar situation at Kansas City, where the maximum difference, -158 μgal was at 11894 B, but at 11894 K and J it was about 100 μgal less. Other big differences were at Houston -146 to -114 μgal. There the local ties were not reasons for differences. A reason might be in changes of environments. After elimination of these large discrepancies all other differences are between +91 and -102 μgal and there seems to be a slight systematic trend, about 120 per 2 million which is a relatively small systematic effect.

In Figure 2 the differences between IGSN 71 and Adjustment E are presented as a function of magnitude of gravity. The same remarks apply as in the above case except the slope is now about 200 per million. The differences between IGSN 71 and Adjustment F are shown in Figure 3. There is hardly any slope, but a clear systematic difference, mean value +33.7 μgal.

A surprising thing is that a slope for differences between Adjustment D and E is about 100 per 2 million. It is questionable if the new observations should be combined with IGSN 71 values at this time.

The new absolute and relative gravity measurements and their careful adjustments and analyses have given us a hope for an improved reference system in the United States. There are few things, which should be done before a reference system much superior to the IGSN 71 can be accomplished. The 100 microgal differences between the absolute measurements at some sites must be resolved. Causes for these differences could be instrumental in nature, but they could also be partly produced by environmental factors, e.g., changing water level, moving air masses, ocean loading, ect. Comparison of absolute gravity apparatuses should be continued as well as studies on the effect of changes in environments on gravity. Meanwhile,

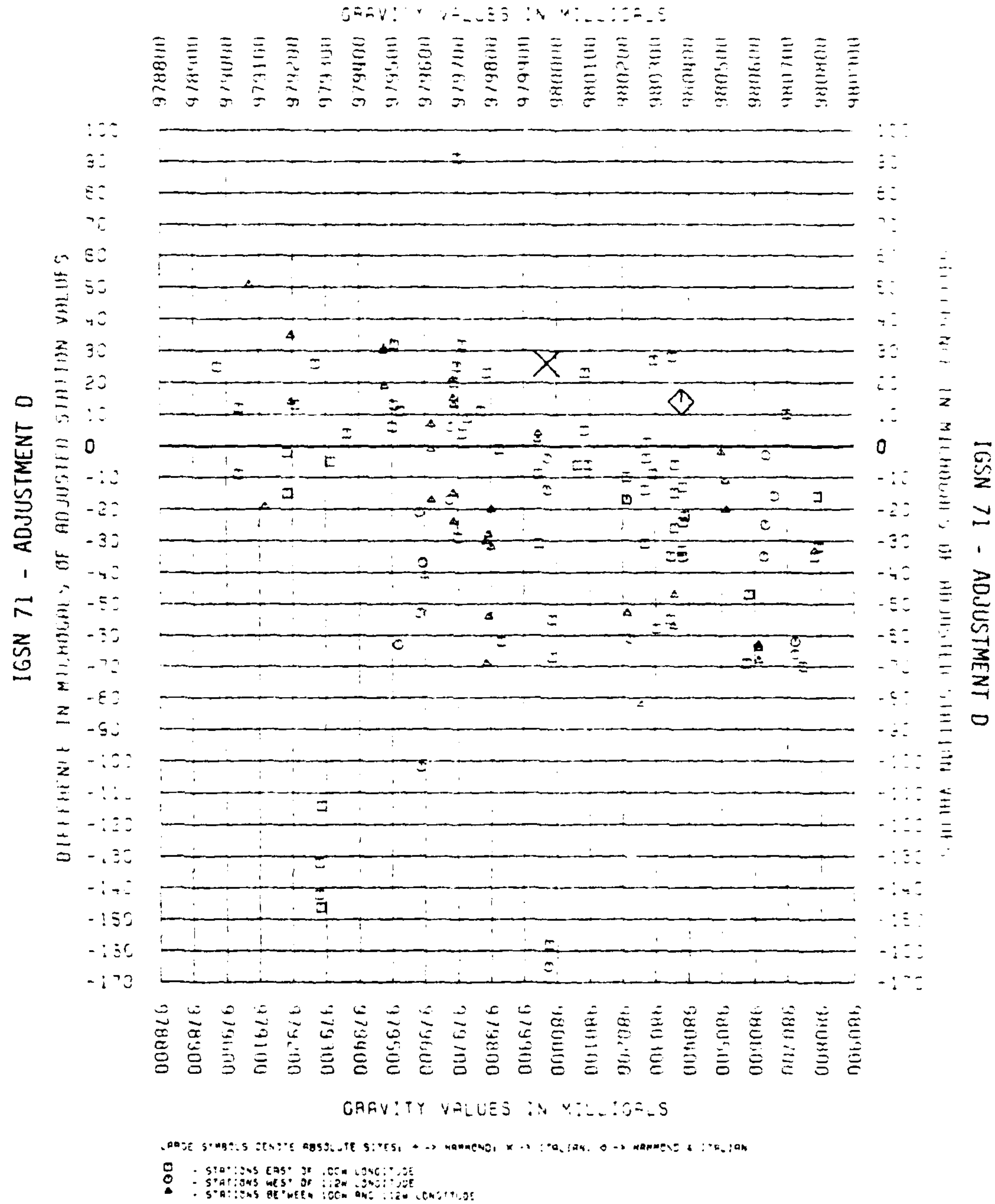
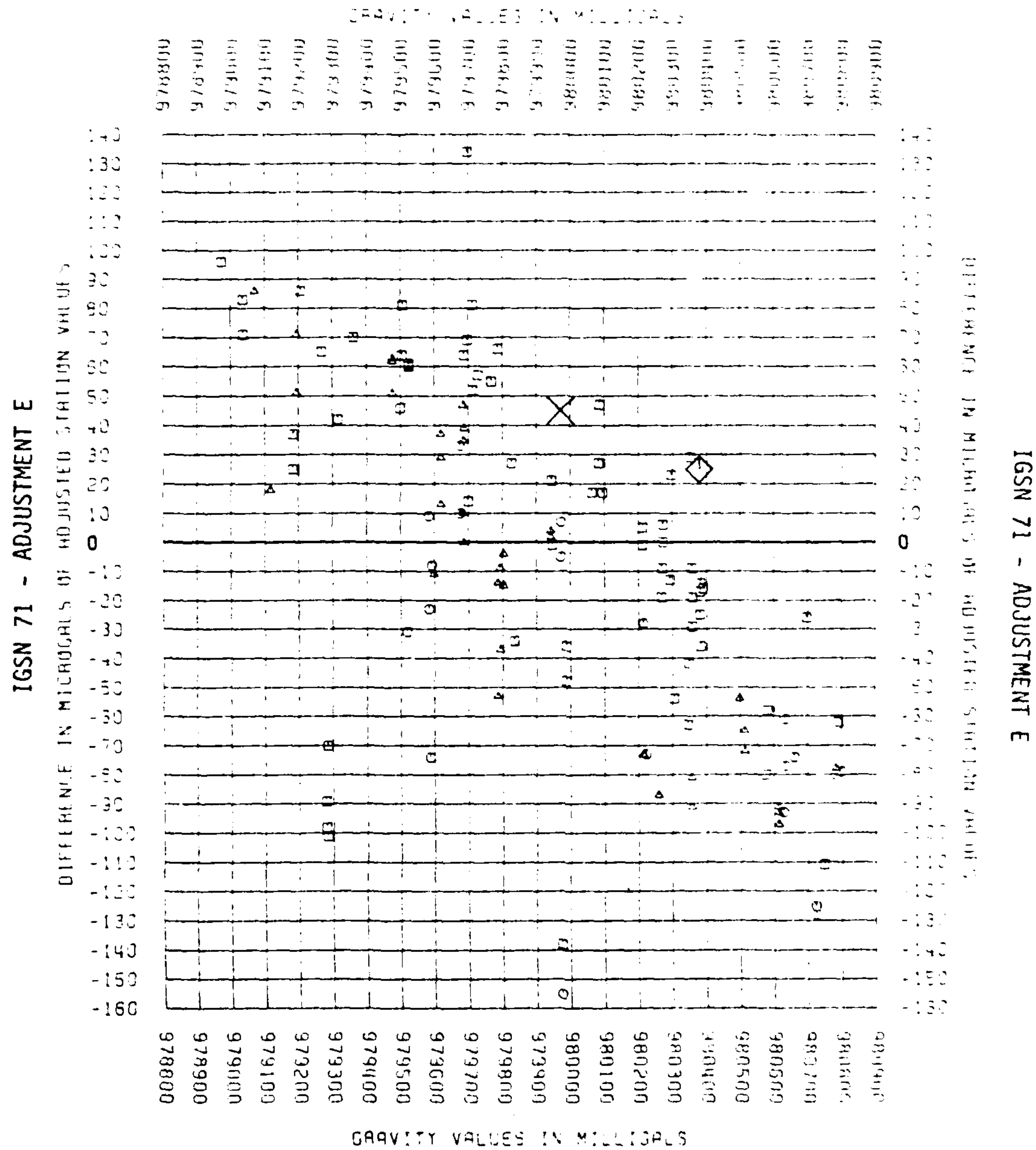


Figure 1

DIFFERENCES BETWEEN IGSN 71 AND ADJUSTMENT D



LARGE SYMBOLS DENOTE ABSOLUTE SITES. + -> HAMMOND, X -> ITALIAN, ◇ -> HAMMOND & ITALIAN

- STATIONS EAST OF 100W LONGITUDE
- STATIONS WEST OF 112W LONGITUDE
▲ - STATIONS BETWEEN 100W AND 112W LONGITUDE

Figure 2

DIFFERENCES BETWEEN IGSN 71 AND ADJUSTMENT E

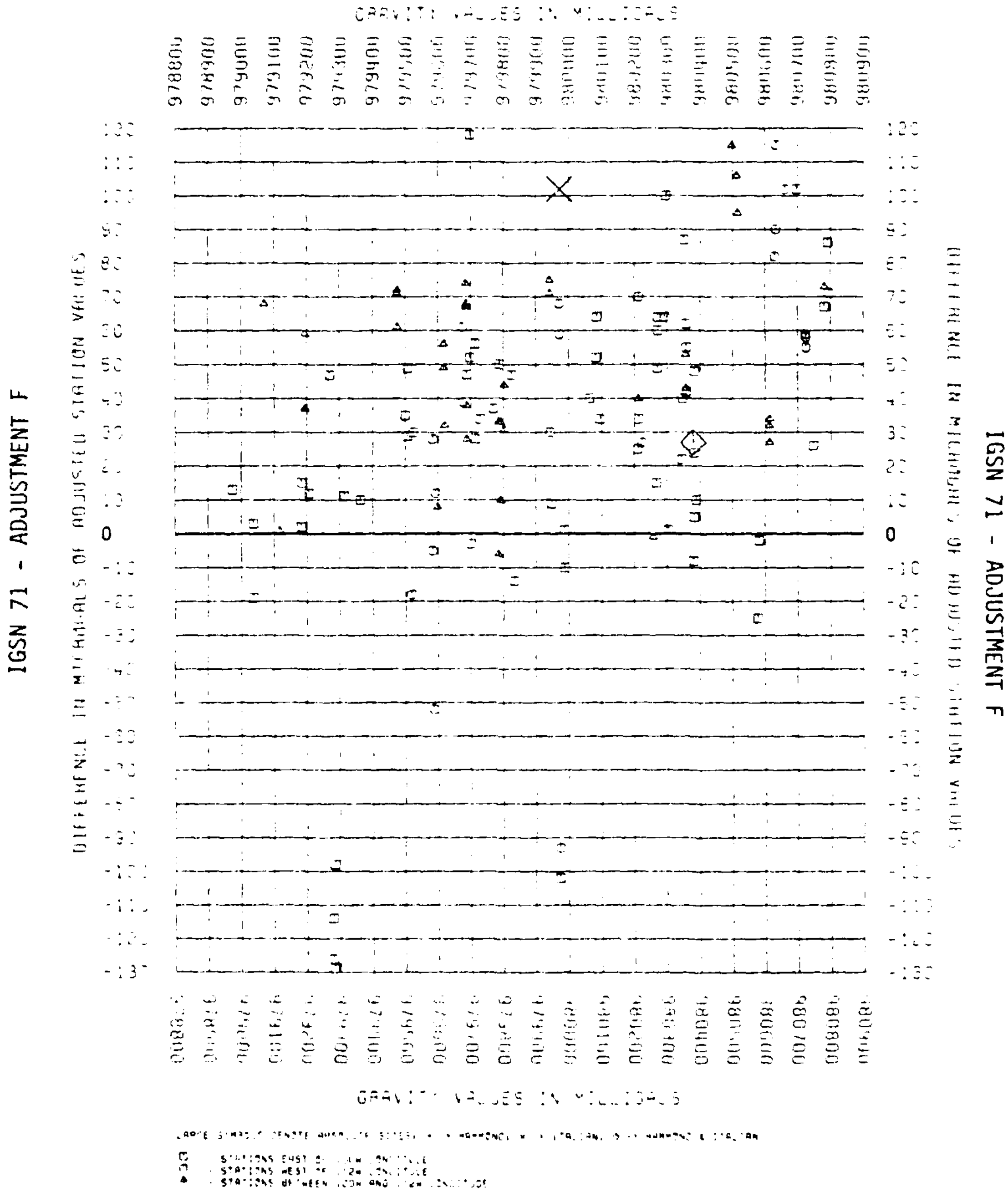


Figure 3

DIFFERENCES BETWEEN IGSN 71 AND ADJUSTMENT F

better calibrations for gravity meters should be accomplished by establishing necessary calibration lines and improving laboratory calibration procedures. In order to strengthen the base network, additional relative gravity measurements are needed. These new measurements should be so scheduled, that most of the instrumental behaviors could be mathematically modeled as well as possible. There will be a new, significantly improved reference system for the United States in the future, but how soon, is difficult to answer.

7. Gravity Induced Position Errors in Inertial Navigation

Klaus-Peter Schwarz in his report [1981] investigates the feasibility of improving airborne inertial navigation by use of gravity field approximations which are more accurate than the normal model presently applied. The effect of the anomalous gravity field on positioning is investigated by Schwarz using a simplified dynamical error model and deriving analytical expressions for the steady state error via the state space approach. In this approach, changes in the anomalous gravity field are cast into the form of first-order differential equations which are related to a position dependent covariance representation of the gravity field by way of the vehicle velocity. Schwarz discusses in his report, different possibilities for a state space model of the covariance model. The procedure chosen by Schwarz combines the consistency of the Tscherning-Rapp model with the analytical advantages of a formulation in terms of Markov processes by making use of the essential parameters of a covariance function proposed by Moritz. The expressions for the gravity induced position errors resulting from this approach are easy to compute for a wide variety of cases. The assumptions made to derive them are in general justifiable.

Based on the available gravity field information a number of approximation models are proposed by Schwarz and expressed in terms of equivalent spherical harmonic expansions. Results show that the use of presently available global models would reduce the gravity induced position errors from σ = 300m to about σ = 150m. Schwarz estimates that improved global models expected in the near future, as for instance those from the GRAVSAT mission, would bring errors below σ = 50 m. However, to reach the meter range, a gravity field appproximation equivalent to an expansion of degree

and order 1000 would be necessary, according to Schwarz. This result is not surprising. It demonstrates the well-known fact that the medium and high frequency spectrum contributes considerably to the deflections of the vertical or, in other words, that the relative contribution of local effects is not negligible in this case. On the other hand, considering the accuracy of present day inertial sensors, accuracies in the meter range may not be realistic for airborne inertial navigation in the near future.

In the last part of his report Schwarz has a brief discussion of different representations of high degree anomalous gravity fields, looking especially at mean values, mass point models, and models with local support at flying altitude. Finally, Schwarz analyses the results with view to their terrestrial applications in inertial survey systems.

8. Mass Point Modeling

Sünkel takes a new look into mass point modeling of gravity related quantities in his report [1981d]. First he analyses the relations of depths of anomalous mass distributions and the gravity covariance functions. He concludes, among other things, that the global gravity anomaly covariance function with a correlation length of about 45 km cannot be generated by mass anomalies, located below 30 km, alone; mass anomalies closer to the surface must be used to present observed 45 km correlation length. The topographic masses and near surface mass anomalies alone can hardly account for this correlation length.

In the next chapter Sünkel discusses discrete point masses and corresponding gravity anomaly degree variance at zero altitude, but he limits his discussions at one constant depth.

Next Sünkel obtains the degree variances as a quadratic form with a positive definite matrix, determined by linear functionals of Legendre polynomials. He states that for a large number of irregularly distributed mass points, the calculation of that quadratic form is a time consuming task; however, the picture changes dramatically if the distribution is regular with respect to a geographical grid. In that case the mathematical apparatus of two-dimensional fast Fourier transformation on the sphere can

be successfully applied to reduce the calculation requirements of the degree variances tremendously; this is the topic of the fourth chapter of Sünkel's report.

For reasons discussed by Sünkel in the first part of his report, mass points cannot be burried at a single depth; several levels should be taken into account. In chapter five Sünkel investigates the impact of the superposition of several point mass layers on the degree variances and the essential parameters of the corresponding covariance function. Particularly instructive is a comparison with covariance models proposed by various authors.

Mathematical problems related to the actual calculations of point masses from observed gravity anomalies has to be studied in the future.

9. List of Reports Produced Under the Contract

Krieg, Lenny A., 1981, The Behavior of the LaCoste and Romberg "G" Gravity Meter, Department of Geodetic Science and Surveying Report No. 321. AFGL-TR-81-0330, Scientific Report No. 10.

Meissl, Peter, 1981, The Use of Finite Elements in Physical Geodesy, Department of Geodetic Science Report No. 313, The Ohio State University, Columbus, Ohio, AFGL-TR-81-0114, Scientific Report No. 7.

Moritz, Helmut, 1979, Concepts in Geodetic Reference Frames, Department of Geodetic Science Report No. 294, The Ohio State University, Columbus, Ohio AFGL-TR-80-0052, Scientific Report No. 2.

Moritz, Helmut, 1980, Theories of Nutation and Polar Motion I, Department of Geodetic Science Report No. 309, The Ohio State University, Columbus, Ohio, AFGL-TR-80-0363, Scientific Report No. 4.

Moritz, Helmut, 1981, Theories of Nutation and Polar Motion II, Department of Geodetic Science and Surveying Report No. 318, The Ohio State University, Columbus, Ohio, AFGL-TR-81-0288, Scientific Report No. 8.

Schwarz, Klaus-Peter, 1981, Gravity induced Position Errors In Inertial Navigation, Department of Geodetic Science and Surveying Report No. 326, The Ohio State State University, AFGL-TR- - , Scientific Report No. . (Awaiting approval from AFGL)

Sünkel, Hans, 1979a, A Covariance Approximation Procedure, Department of Geodetic Science Report No. 286, The Ohio State University, Columbus, Ohio, AFGL-TR-0075, Scientific Report No. 1.

Sünkel, Hans, 1979b, A General Surface Representation Module Designed for Geodesy, Department of Geodetic Science Report No. 292, The Ohio State University, Columbus, Ohio, AFGL-TR-80-0204, Scientific Report No. 3.

Sünkel, Hans, 1980a, Program Listing for the General Surface Representation Module Designed for Geodesy (GSPP), The Ohio State University, Columbus, Ohio. Internal Report.

Sünkel, Hans, 1980b, Manual to the General Surface Representation Module Designed for Geodesy (GSPP), The Ohio State University, Columbus, Ohio. Internal report.

Sünkel, Hans, 1981a, Feasibility Studies for the Prediction of the Gravity Disturbance Vector in High Altitudes, Department of Geodetic Science Report No. 311, The Ohio State University, Columbus, Ohio, AFGL-TR-81-0084, Scientific Report No. 5.

Sünkel, Hans, 1981b, Cardinal Interpolation, Department of Geodetic Science Report No. 312, The Ohio State University, Columbus, Ohio, AFGL-TR-81-0107, Scientific Report No. 6.

Sünkel, Hans, 1981c, The Estimation of Free-Air Anomalies, Department of Geodetic Science and Surveying Report No. 315, The Ohio State University, Columbus, Ohio, AFGL-TR-81-0029, Scientific Report No. 9.

Sünkel, Hans, 1981d, Point Mass Models of the Anomalous Gravitational Field, Department of Geodetic Science and Surveying Report No. 328, The Ohio State University, Columbus, Ohio, AFGL-TR- - , Scientific Report No. . (Awaiting approval from AFGL)

10. List of Scientific Personnel

Principal Investigator

Urho A. Uotila

Research Associates and Consultants

Peter Meissl

Helmut Moritz

Klaus-Peter Schwarz

Hans Sünkel

Graduate Research Associates

Edward M. Baker

Mohammed S.A. Hanafy

Lenny A. Krieg

Raul P. Ramirez

Administrative Secretary

Roberta A. Canegali

DATE
ILME